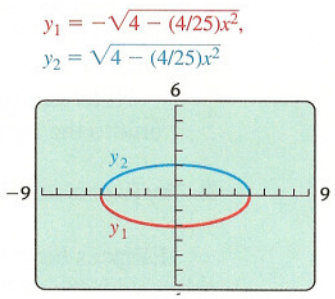
Graphing Ellipses

Graphing an ellipse on a graphing calculator is much like graphing a circle: We graph it in two pieces after solving for y.

To graph the ellipse given by the equation $4x^2 + 25y^2 = 100$, we first solve for y:

$$\begin{aligned} 4x^2 + 25y^2 &= 100 \\ 25y^2 &= 100 - 4x^2 \\ y^2 &= 4 - \tfrac{4}{25}x^2 \\ y &= \pm\sqrt{4 - \tfrac{4}{25}x^2}. \end{aligned}$$

Then, using a squared window, we graph.



On many calculators, pressing APPS and selecting Conics and then Ellipse accesses a program in which equations in standard form of ellipses centered a (h, k) can be graphed directly.

Your Turn

Graph the ellipse $9x^2 + y^2 = 36$ using the following steps.

1. Solve $9x^2 + y^2 = 36$ for y. $y = \pm\sqrt{36 - 9x^2}$
2. Enter $y_1 = \sqrt{36 - 9x^2}$ and $y_2 = -y_1$.
3. Set up a $[-9, 9, -6, 6]$ window and graph both equations.

10.2 Exercise Set

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Concept Reinforcement *Classify each of the following statements as either true or false.*

1. The graph of $\frac{x^2}{28} + \frac{y^2}{48} = 1$ is a vertical ellipse. True
2. The graph of $\frac{x^2}{30} + \frac{y^2}{20} = 1$ is a vertical ellipse. False
3. The graph of $\frac{x^2}{25} - \frac{y^2}{9} = 1$ is a horizontal ellipse. False
4. The graph of $\frac{-x^2}{20} + \frac{y^2}{16} = 1$ is a horizontal ellipse. False
5. The graph of $\frac{x^2}{9} + \frac{y^2}{25} = 1$ includes the points $(-3, 0)$ and $(3, 0)$. True
6. The graph of $\frac{x^2}{36} + \frac{y^2}{25} = 1$ includes the points $(0, -5)$ and $(0, 5)$. True

7. The graph of $\frac{(x+3)^2}{25} + \frac{(y-2)^2}{36} = 1$ is an ellipse centered at $(-3, 2)$. True

8. The graph of $\frac{(x-2)^2}{49} + \frac{(y+5)^2}{9} = 1$ is an ellipse centered at $(2, -5)$. True

Graph each of the following equations.

9. $\frac{x^2}{1} + \frac{y^2}{9} = 1$ ⊡

10. $\frac{x^2}{9} + \frac{y^2}{1} = 1$ ⊡

11. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ ⊡

12. $\frac{x^2}{16} + \frac{y^2}{25} = 1$ ⊡

13. $4x^2 + 9y^2 = 36$ ⊡

14. $9x^2 + 4y^2 = 36$ ⊡

15. $16x^2 + 9y^2 = 144$ ⊡

16. $9x^2 + 16y^2 = 144$ ⊡

17. $2x^2 + 3y^2 = 6$ ⊡

18. $5x^2 + 7y^2 = 35$ ⊡

Aha! 19. $5x^2 + 5y^2 = 125$ ⊡

20. $8x^2 + 5y^2 = 80$ ⊡

21. $3x^2 + 7y^2 - 63 = 0$ ⊡

22. $3x^2 + 8y^2 - 72 = 0$ ⊡

23. $16x^2 = 16 - y^2$ ⊡

24. $9y^2 = 9 - x^2$ ⊡

25. $16x^2 + 25y^2 = 1$ ⊡

26. $9x^2 + 4y^2 = 1$ ⊡

27. $\frac{(x-3)^2}{9} + \frac{(y-2)^2}{25} = 1$ ⊡

28. $\frac{(x-2)^2}{25} + \frac{(y-4)^2}{9} = 1$ ⊡

29. $\frac{(x+4)^2}{16} + \frac{(y-3)^2}{49} = 1$ ⊡

30. $\frac{(x+5)^2}{4} + \frac{(y-2)^2}{36} = 1$ ⊡

⊡ Answers to Exercises 9–34, 47, and 48 are on p. IA-28.

31. $12(x-1)^2 + 3(y+4)^2 = 48$ (*Hint*: Divide both sides by 48.) ⊡

32. $4(x-6)^2 + 9(y+2)^2 = 36$ ⊡

Aha! 33. $4(x+3)^2 + 4(y+1)^2 - 10 = 90$ ⊡

34. $9(x+6)^2 + (y+2)^2 - 20 = 61$ ⊡

TW 35. Explain how you can tell from the equation of an ellipse whether the graph will be horizontal or vertical.

TW 36. Can an ellipse ever be the graph of a function? Why or why not?

SKILL REVIEW

Review solving equations.

Solve.

37. $x^2 - 5x + 3 = 0$ [8.2] $\frac{5}{2} \pm \frac{\sqrt{13}}{2}$

38. $\log_x 81 = 4$ [9.6] 3

39. $\frac{4}{x+2} + \frac{3}{2x-1} = 2$ [6.4] $-\frac{3}{4}, 2$

40. $3 - \sqrt{2x-1} = 1$ [7.6] $\frac{5}{2}$

41. $x^2 = 11$ [8.1] $-\sqrt{11}, \sqrt{11}$

42. $x^2 + 4x = 60$ [5.4] $-10, 6$

SYNTHESIS

TW 43. Explain how it is possible to recognize that the graph of $9x^2 + 18x + y^2 - 4y + 4 = 0$ is an ellipse.

TW 44. As the foci get closer to the center of an ellipse, what shape does the graph begin to resemble? Explain why this happens.

Find an equation of an ellipse that contains the following points.

45. $(-9, 0)$, $(9, 0)$, $(0, -11)$, and $(0, 11)$ $\frac{x^2}{81} + \frac{y^2}{121} = 1$

46. $(-7, 0)$, $(7, 0)$, $(0, -5)$, and $(0, 5)$ $\frac{x^2}{49} + \frac{y^2}{25} = 1$

47. $(-2, -1)$, $(6, -1)$, $(2, -4)$, and $(2, 2)$ ⊡

48. $(4, 3)$, $(-6, 3)$, $(-1, -1)$ and $(-1, 7)$ ⊡

49. *Theatrical Lighting.* The spotlight on a violin soloist casts an ellipse of light on the floor below her that is 6 ft wide and 10 ft long. Find an equation of that ellipse if the performer is in its center, x is the distance from the performer to the side of the ellipse, and y is the distance from the performer to the top of the ellipse. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

50. *Astronomy.* The maximum distance of the planet Mars from the sun is 2.48×10^8 mi. The minimum distance is 3.46×10^7 mi. The sun is at one focus of the elliptical orbit. Find the distance from the sun to the other focus. 2.134×10^8 mi

51. Let $(-c, 0)$ and $(c, 0)$ be the foci of an ellipse. Any point $P(x, y)$ is on the ellipse if the sum of the distances from the foci to P is some constant. Use $2a$ to represent this constant.

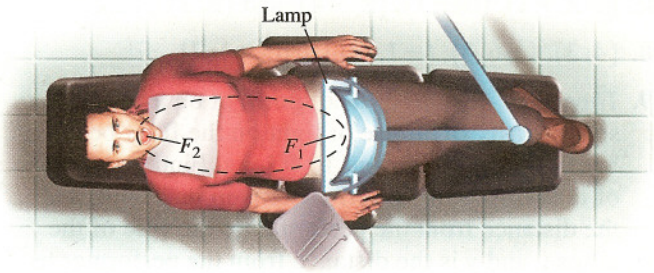
a) Show that an equation for the ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1. \quad \square$$

b) Substitute b^2 for $a^2 - c^2$ to get standard form. $\square$

52. *President's Office.* The Oval Office of the President of the United States is an ellipse 31 ft wide and 38 ft long. Show in a sketch precisely where the President and an adviser could sit to best hear each other using the room's acoustics. (*Hint*: See Exercise 51(b) and the discussion following Example 3.) $\square$

53. *Dentistry.* The light source in a dental lamp shines against a reflector that is shaped like a portion of an ellipse in which the light source is one focus of the ellipse. Reflected light enters a patient's mouth at the other focus of the ellipse. If the ellipse from which the reflector was formed is 2 ft wide and 6 ft long, how far should the patient's mouth be from the light source? (*Hint*: See Exercise 51(b).) 5.66 ft



54. *Firefighting.* The size and shape of certain forest fires can be approximated as the union of two "half-ellipses." For the blaze modeled below, the equation of the smaller ellipse—the part of the fire moving *into* the wind—is

$$\frac{x^2}{40{,}000} + \frac{y^2}{10{,}000} = 1.$$

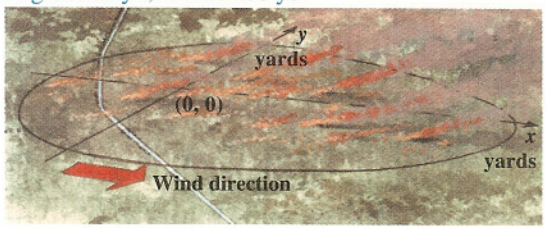
The equation of the other ellipse—the part moving *with* the wind—is

$$\frac{x^2}{250{,}000} + \frac{y^2}{10{,}000} = 1.$$

Determine the width and the length of the fire.

Source for figure: "Predicting Wind-Driven Wild Land Fire Size and Shape," Hal E. Anderson, Research Paper INT-305, U.S. Department of Agriculture, Forest Service, February 1983

Length: 700 yd; width: 200 yd



For each of the following equations, complete the square as needed and find an equivalent equation in standard form. Then graph the ellipse.

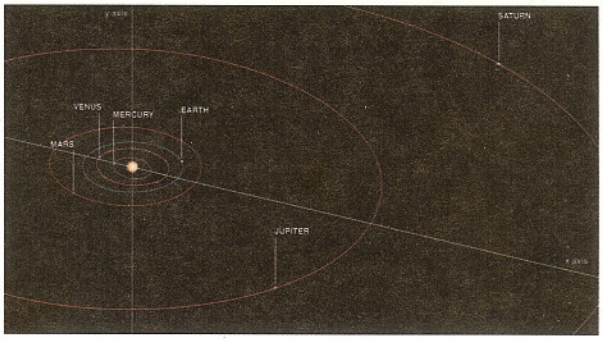
55. $x^2 - 4x + 4y^2 + 8y - 8 = 0$ $\square$

56. $4x^2 + 24x + y^2 - 2y - 63 = 0$ $\square$

Astronomy. The earth's orbit around the sun is an ellipse with $a \approx 149.7$ million km. The sun, located at one focus of the ellipse, is approximately 2.4 million km from the center of the ellipse.

Source: Based on information from Physical Geography.net

57. What is the maximum distance of the earth from the sun? 152.1 million km



$\square$ Answers to Exercises 51, 52, 55, and 56 are on pp. IA-28 and IA-29.